Written Exam at the Department of Economics winter 2017-18

Economics of the Environment, Natural Resources and Climate Change

Final Exam

12 January, 2018

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam question consists of 5 pages in total, including the front page.

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

EXERCISE 1. Optimal natural resource extraction with pollution

Consider a model of the economy and the environment that uses the following notation:

$Y =$ production of final goods
$K =$ stock of man-made capital (physical and human)
$R =$ input of an exhaustible natural resource (raw material)
$S =$ reserve stock of the natural resource
$E =$ total emission of pollutant
$b =$ emission of pollutant per unit of raw material used in final goods production
$a =$ cost of extracting one unit of the natural resource (measured in units of the final good)
$C =$ consumption of final goods
$I =$ investment in produced capital
$U =$ lifetime utility of the representative consumer
$u =$ flow of utility per period
$\rho =$ rate of time preference
$t =$ time (treated as a continuous variable)

The lifetime utility of the representative consumer at time zero is

$$U_0 = \int_0^{\infty} u\left(C_t\right) e^{-\rho t} dt, \qquad u' > 0, \quad u'' < 0, \quad \rho > 0. \tag{1}$$

In the following, all variables except the constant parameters a, b and ρ will be understood to be functions of time, so for convenience we will generally skip the time subscripts.

The production of final goods is given by the production function

$$Y = F\left(K, R, E\right), \qquad F_K \equiv \frac{\partial F}{\partial K} > 0, \quad F_R \equiv \frac{\partial F}{\partial R} > 0, \quad F_E \equiv \frac{\partial F}{\partial E} < 0. \tag{2}$$

The assumption that $F_E < 0$ reflects that pollution has a negative impact on productivity.

Pollution is caused by the transformation of the raw material in the process of production. The use of one unit of raw material generates an emission of b units of the pollutant, so total emissions are

$$E = bR, \qquad b \text{ constant.} \tag{3}$$

The total cost of raw material production (measured in units of final goods) is aR, where a is the cost of extracting one unit of the natural resource. Hence the economy's aggregate resource constraint is

$$Y = C + I + aR, \qquad a \text{ constant.} \tag{4}$$

We will abstract from depreciation, so the net investment in man-made capital is equal to the gross investment I. Hence the change over time in the capital stock is

$$\dot{K} \equiv \frac{dK}{dt} = I. \tag{5}$$

We also abstract from the discovery of new reserves of the natural resource, so the change over time in the exhaustible resource stock is

$$\dot{S} \equiv \frac{dS}{dt} = -R. \tag{6}$$

The initial stocks of man-made and natural capital (K_0 and S_0) are predetermined.

Our first task is to characterize the first-best optimal allocation of resources that would be chosen by a benevolent social planner who maximizes the utility function (1) subject to the constraints implied by eqs. (2) through (6), taking K_0 and S_0 as given.

Question 1.1: Show that the current-value Hamiltonian corresponding to the social planner's problem may be written as

$$H = u\left(C\right) + \mu\left[F\left(K,R,bR\right) - C - aR\right] - \lambda R, \tag{7}$$

where μ is the shadow value of K, and λ is the shadow value of S. What are the control variables and what are the state variables in the social planner's optimal control problem?

Question 1.2: Derive the first-order conditions for the solution to the social planner's optimal control problem.

Question 1.3: Show that the first-order conditions for the solution to the social planner's problem imply the following condition for an optimal exploitation of the natural resource:

$$\dot{F}_R + b\dot{F}_E = \left(F_R + bF_E - a\right)F_K. \tag{8}$$

Give an economic interpretation of eq. (8) and explain the economic intuition behind it.

We will now consider the resource allocation that will materialize in a market economy with private property and perfect competition in all markets. We start by focusing on the production of raw materials, and we assume that the reserves of natural resources are owned by private mining firms. The market value of the representative mining firm at time zero is denoted by V_0^R, and the firm's payout of net dividends in the future period t is denoted by D_t^R. The market value of the firm is the present value of the future net dividends paid out to its owners, that is

$$V_0^R = \int_0^{\infty} D_t^R e^{-\int_0^t r_s ds} dt, \tag{9}$$

where r is the real market interest rate which may vary over time. The market price of the raw material (measured relative to the price of final goods) is p which also varies over time. The net dividend paid out by the mining firm in period t is therefore given by

$$D_t^R = (p_t - a) R_t. \tag{10}$$

The mining firm chooses its rate of raw material extraction R so as to maximize its market value (9) subject to (10), taking the price p as given, and accounting for the stock-flow relationship (6) between its current rate of extraction and its remaining reserve stock of the resource.

Question 1.4: Set up the current-value Hamiltonian for the mining firm's optimal control problem (you may denote the shadow price of the reserve stock by λ^R, and for convenience you may skip the time subscripts).

Question 1.5: Derive the first-order conditions for the solution to the mining firm's problem.

Question 1.6: Show that the mining firm's first-order conditions imply that

$$\dot{p} = (p - a) r. \tag{11}$$

Give an economic interpretation of eq. (11) and explain the economic intuition behind it.

Consider next the representative firm producing the final good Y which we use as our numeraire good, thus setting its price equal to 1. The technology of the final goods firm is given by the production function (2), but since the representative firm is small relative to the market, it takes the aggregate flow of pollution E as given. The firm thus neglects its own contribution to total emissions and the resulting negative impact on the productivity of all firms. Its market value at time zero (V_0^Y) is the present value of its future net dividends,

$$V_0^Y = \int_0^{\infty} D_t^Y e^{-\int_0^t r_s ds} dt, \tag{12}$$

where D_t^Y is the net dividend paid out in the future period t. The government levies an environmental tax at the rate τ_t per unit of raw material used in final goods production, so the firm's total cost of a unit of raw material is $p_t + \tau_t$. Hence the net dividend paid out by the final goods firm is

$$D_t^Y = F(K_t, R_t, E_t) - (p_t + \tau_t) R_t - I_t. \tag{13}$$

The final goods firm chooses R and I with the purpose of maximizing its market value (12) subject to (13), taking the aggregate emission flow E as given, and accounting for the stock-flow relationship (5) between its investment and the change in its capital stock.

Question 1.7: Set up the current-value Hamiltonian for the optimal control problem of the final goods firm (you may denote the shadow price of its capital stock by μ^Y, and for convenience you may skip the time subscripts).

Question 1.8: Derive the first-order conditions for the solution to the problem of the final goods firm. Show that these conditions imply that

$$F_K = r, \tag{14}$$

$$\dot{p} = \dot{F}_R - \dot{\tau}. \tag{15}$$

Question 1.9: Use your findings in Question 1.6 and Question 1.8 to derive an expression for the optimal environmental tax rate τ which will ensure that resource extraction in the market economy will satisfy the condition (8) for a socially optimal rate of extraction. Explain the economic intuition for your result.

EXERCISE 2. Green tax reform

(Note: The questions in this exercise may be answered without any use of math and/or graphical analysis. However, you are welcome to use math or diagrams to the extent that you find it convenient).

In a "green" tax reform the government introduces (or raises) taxes on polluting goods and uses the revenue to lower taxes on income. In the following, you may assume that the green tax reform involves the introduction of a tax on one polluting good and that the revenue is used to lower a proportional tax on labour income.

Question 2.1: Explain the effects on economic efficiency (welfare) of a green tax reform (Hint: Is there a positive "second dividend"?)

Question 2.2: Will it be (second-best) optimal for a green tax reform to set the tax rate on the polluting good equal to the Pigouvian tax rate? Motivate your answer.